TERM- 1 Activity No. 1, 3, 6, 10, 15

TERM- 2 Activity No. 20, 21, 23, 26, 27

TERM-1

ACTIVITY 1

OBJECTIVE:

To verity that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m): l \perp m\}$ is symmetric but neither reflexive nor transitive.

PRE-REQUIRED KNOWLEDGE:

Knowledge of relations & properties of parallel lines and perpendicular lines.

MATERIALS REQUIRED:

A thick board, graph papers, drawing pins, ruler, pencil, glue, colour pens etc.

PROCEDURE:

(1) Take a thick board attached a graph paper on it with the help of drawing pins as shown in fig 1(a).

(2) Draw two parallel lines q_1 & q_2 with blue colour pen& draw three perpendicular lines q_3, q_4 & q_5 with green colour pen.

(3) Similarly, draw two more parallel lines q_6 & q_7 & a perpendicular line q_8 with red colour pen.

(4) Since, line q_3 is perpendicular to q_1 & q_2, line q_4 is perpendicular to q_1 & q_2 and q_5 is perpendicular to q_1 & q_2. Also, line q_8 is perpendicular on lines q_6 & q_7 as shown in figure 1 (b).

(5) Since, lines q_1 & q_2 are parallel to each other and q_3 is parallel to q_4, q_4 is parallel to q_5 & q_5 is parallel to q_3. Also, q_6 is parallel to q_7.

(6) So, $(q_3, q_1), (q_3, q_2), (q_4, q_1), (q_4, q_2), (q_5, q_1), (q_5, q_2), (q_8, q_6), (q_8, q_7)$ R.

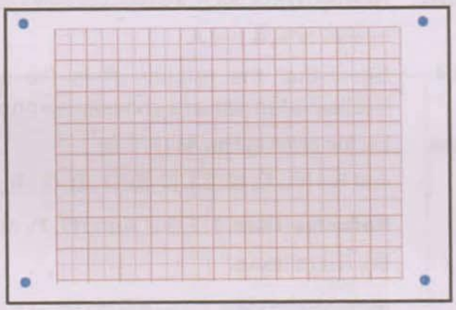

Figure 1 (a)

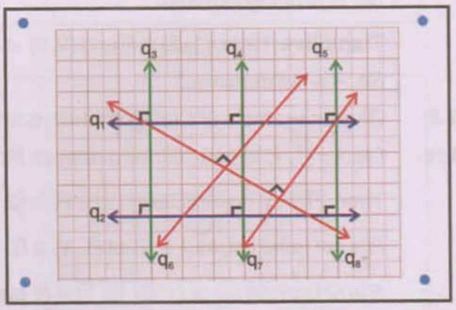

Figure 1 (b)

OBSERVATIONS:

(1) In figure 1 (b), we see that $q_3 \perp q_1$. Then, $q_1 \perp q_3$ i.e. $(q_3, q_1) \in R \Rightarrow (q_1, q_3) \in R$. Similarly, $(q_1, q_4) \in R \Rightarrow (q_4, q_1) \in R$ & (q_7, q_8) $R \Rightarrow (q_8, q_7)$ R.

So, the given relation R is symmetric.

(2) Since, "No line is perpendicular to itself" or $(l, l) \notin R$

So, the given relation R is not reflexive.

i.e. $R = \{(l, m) : l \perp m\}$ is not reflexive.

(3) It is also observed that $q_3 \perp q_1$ & $q_1 \perp q_4$. But, q_3 is $\perp$ not on q_4 i.e. $(q_3, q_1) \in R$ & $(q_1, q_4) \in R$. But, $(q_3, q_4) \notin R$.

So, the given relation R is not transitive.

RESULT:

From the above activity, it is verified that the relation R is the set L of all lines is a plane, defined by $R = \{(l, m): l \perp m\}$ **is symmetric but neither reflexive nor transitive** i.e. the relation **is not an equivalence relation.**

ACTIVITY 3

OBJECTIVE :

To demonstrate a function which is not one-one but is onto.

PRE-REQUIRED KNOWLEDGE :

Basic knowledge of relations, functions and their types (one-one functions and onto functions).

MATERIALS REQUIRED :

A thick board, coloured chart papers, a pair of scissors, gluestick, some board pins, some pieces of thread.

PROCEDURE :

(1) Take a blue chart paper, cut a rectangular piece of length 18 cm and width 5 cm as shown in fig. 3(a).

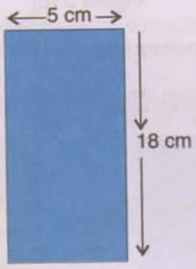

Figure 3 (a)

(2) Take a yellow chart paper, cut a rectangular piece of length 15 cm and width 4 cm as shown in Fig. 3(b).

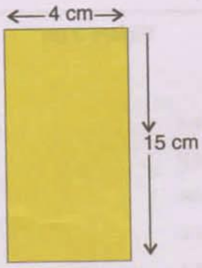

Figure 3 (b)

(3) Paste these two pieces on a thick board and mark these as points P &Q respectively as shown in Fig.3(c).

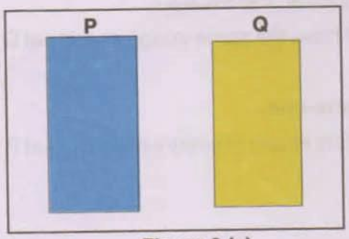

Figure 3 (c)

(4) Fix five drawing pins on the blue paper and mark the pins as a, b, c, d & e. Similarly, fix three drawing pins on yellow paper and mark them as x, y and z as shown in Fig.3 (d) i.e. P = {a, b, c, d, e} & Q = {x, y, z}

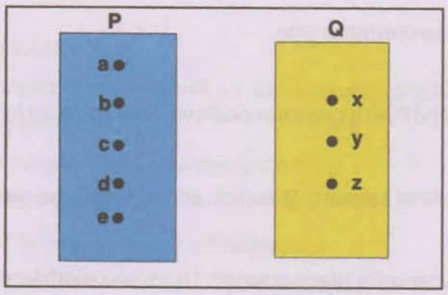

Figure 3 (d)

(5) Join these drawing pins with the help of threads as shown in Fig.3(e) i.e. join the elements of P to the elements of Q.

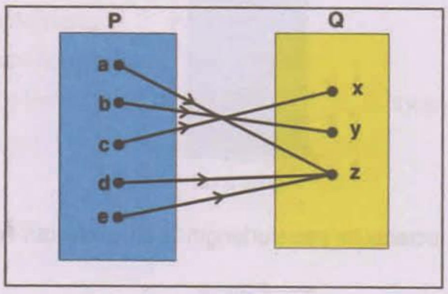

Figure 3 (e)

OBSERVATIONS :

(1) The image of the element a of set P in set Q is z.

(2) The image of the element b of set P in set Q is y.

(3) The image of the element c of set P in set Q is x.

(4) The image of the element d of set P in set Q is z.

(5) The image of the element e of set P in set Q is z.

(6) The pre-image of the element x of set Q in set P is c.

(7) The pre-image of the element y of set Q in set P is b.

(8) The pre-image of the element z of set Q in set P are a, d & e.

(9) Since, the elements of a, d & e in set P have the same image as z in set Q i.e. every element of set P has not one-one image in set Q.

So, the function is many-one or not one-one.

(10) Also, we see that every element of set Q is image of every element of set P i.e. the pre-image of every element of Q in P exists.

So, the function is onto.

RESULT :

From the above activity we can say that the function is **not one-one but onto.**

ACTIVITY 6

OBJECTIVE :

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

PRE-REQUIRED KNOWLEDGE :

Basic knowledge & properties of trigonometric ratios and inverse trigonometric functions.

MATERIALS REQUIRED :

A thick board, white paper, drawing pins, two sticks, ruler, needle and wires.

PROCEDURE :

(1) Take a white paper on a thick board with the help of drawing pins as shown in Fig.6(a).

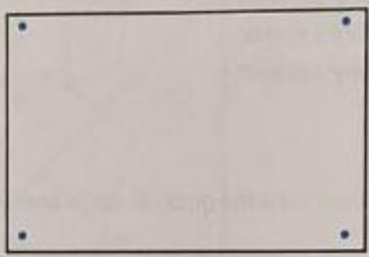

Figure 6 (a)

(2) Take radius = 1 unit and draw a circle with centre O on it.

(3) Through the centre of the circle draw perpendicular axes XOX' and YOY' as x-axis and y-axis respectively as shown in Fig. 6(b).

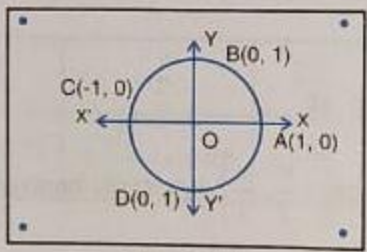

Figure 6 (b)

(4) Mark the points A, B, C and D, where the circle cuts the x-axis and y-axis respectively as shown in Fig. 6(b).

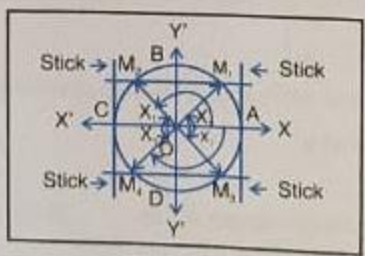

Figure 6 (c)

(5) Fix two sticks parallel to y-axis on the opposite sides of the board as shown in Fig.6(c).

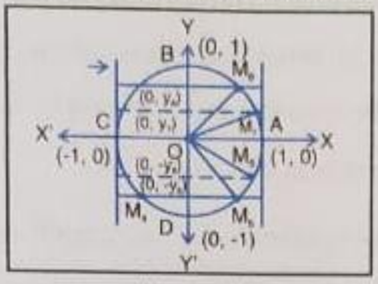

Figure 6 (d)

(6) Now fix one wire between the sticks so that the wire can move freely parallel to the x-axis as shown in Fig. 6(c).

(7) Now fix a needle of unit length such that one end of the needle is at the centre of the circle and the other end to move freely along the circle as shown in Fig. 6(c).

(8) Place the needle at an arbitrary angle x, with the positive x-axis.

(9) Measurement of the angle in radian is equal to the length of intercepted arc of the circle.

(10) Slide the wire between the sticks such that the wire meets with free end of the needle. Mark this point as M_1, as shown in Fig.6(c).

(11) Identify the y-coordinate of point m_1, which is equal to the perpendicular distance from the x-axis. It gives $y_1 = \sin x_1$.

(12) Further rotate the needle in the anticlockwise direction and keep it at the angle $\pi - x_1$. Here the wire meets the needle at point M_2 as shown in Fig.6(c).

(13) With the help of sliding wire, find the value of y-coordinate of M_2.

(14) For both the points M_1 and M_2 the value of y-coordinates are same for different values of angles i.e. y_1 = six x_1 and $y_1 = \sin(\pi - x_1)$.

(15) The above observations show that the sine function is not one-to-one for angles which lie in first and second quadrants.

(16) Repeat the same process for angles $-x_1$ and $(-\pi + x_1)$ respectively. In this case also we will get the same result that y-coordinate for the points M_3 and M_4 are the same. Thus, the sine function is not one-to one for angles that fall in third and fourth quadrant, as shown in Fig.6(c).

(17) Here, we observe that the value of y-coordinate is different for points M_3 and M_1.

(18) Now move the needle again in anti clockwise direction from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$. By sliding the wire, we see that the value of y-coordinate for points M_5, M_6, M_7 and M_8 are different. Therefore, sine function is one -one in the domain $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and thus its range lies between −1 and 1, as shown in Fig.6(d).

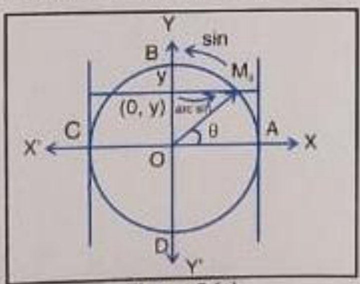

Figure 6 (e)

(19) Now place the needle at any arbitrary angle θ lying in the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and find the y-coordinate of the intersecting point M_9 through the sliding wire. We call it as y.

$y = \sin\theta$ or $\theta = \sin^{-1} y$ as sine function is one-one and onto in the domain $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and range [–1, 1].

Hence, its inverse arc sine function exists as shown in Fig. 6(e).

(20) The domain and range of sine inverse function are interchanged with the domain and range of sine function, i.e., the domain of arc sine function is [–1, 1] and range is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

This range of arc sine function is known as principal value of arc sine function or $\sin^{-1}$ function.

OBSERVATIONS :

(1) Sine function is non-negative in first and second quadrants.

(2) For the third and fourth quadrants, the sine function is negative.

(3) $\theta = \text{arc} \sin y \Rightarrow y = \sin^{-1}\theta$, where $\left[\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}\right]$

RESULT :

From the above activity, we find that the principal value range of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

ACTIVITY 10

OBJECTIVE :

To verify that for a function f to be continuous at given point x_0, $\Delta y = |f(x_0 + \Delta x) - f(x_0)|$ is arbitrarily small provided, Δx is sufficiently small.

PRE-REQUIRED KNOWLEDGE :

Knowledge of continuity of a function.

MATERIALS REQUIRED :

Thick board, white sheets, graph paper, scale, pencil, calculator and gluestick.

PROCEDURE :

(1) On the thick board, paste a graph paper.

(2) Draw the curve of any continuous function $y = f(x)$ as represented in figure 10 (a).

(3) Take a point A(1, 0) on the positive side of x-axis and corresponding to this point, mark the point M(1, 1.4) on the curve.

(4) Take one more point K_1 (1+0.8, 0) or K_1 (1.8, 0) to the right of A. Here, 0.8 is an increment in x.

(5) Draw the perpendicular from K_1 to meet the curve at L_1. The coordinates of L_1 are (1.8, 1.7).

(6) Draw the perpendicular from the point M to meet K_1L_1 at T_1.

(7) Measure AK_1 and L_1T_1.

(8) Repeat these steps by taking one more point K_2 (1 + 0.6, 0) or K_2 (1.6, 0).

(9) Similarly, take points K_3, K_4 and K_5 and locate the corresponding points L_3, L_4 and L_5.

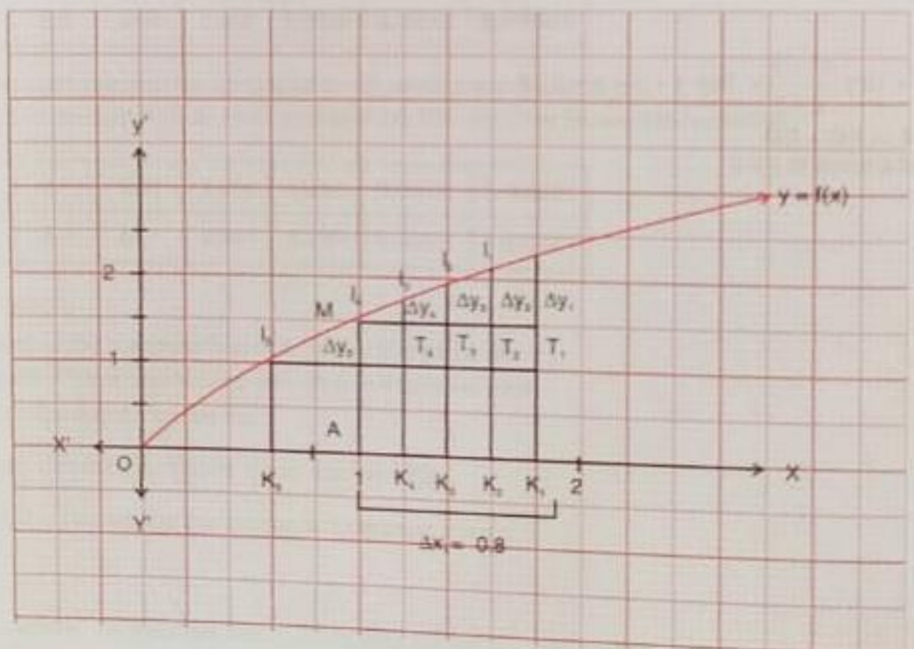

Figure 10

OBSERVATIONS :

(1) From the graph we have the following values.

S.No.	Value of increment in $x_0 (x_0 = 1)$	Corresponding increment in y
1	$\lvert \Delta x_1 \rvert = 0.8$	$\lvert \Delta y_1 \rvert = 0.3$
2	$\lvert \Delta x_2 \rvert = 0.6$	$\lvert \Delta y_2 \rvert = 0.22$
3	$\lvert \Delta x_3 \rvert = 0.4$	$\lvert \Delta y_3 \rvert = 0.14$
4	$\lvert \Delta x_4 \rvert = 0.2$	$\lvert \Delta y_4 \rvert = 0.1$
5	$\lvert \Delta x_5 \rvert = 0.4$	$\lvert \Delta y_5 \rvert = 0.4$

(2) So, Δy becomes smaller when Δx becomes smaller.

(3) Thus, $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

RESULT:

From the above activity, it is verified that for a function to be continuous at any point x_0, $\Delta y = \lvert f(x_0 + \Delta x) - f(x_0) \rvert$ is arbitrarily small, provided Δx is sufficiently small.

ACTIVITY 15

OBJECTIVE :

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

PRE-REQUIRED KNOWLEDGE :

Knowledge of maxima, minima, absolute maxima &absolute minimum.

MATERIALS REQUIRED :

Thick board, white chart paper, sketch pens, calculator, glue stick etc.

PROCEDURE :

(1) Paste a white chart paper of on the thick board.

(2) Draw two lines on the graph paper representing x-axis and y-axis as shown in the Fig.15.

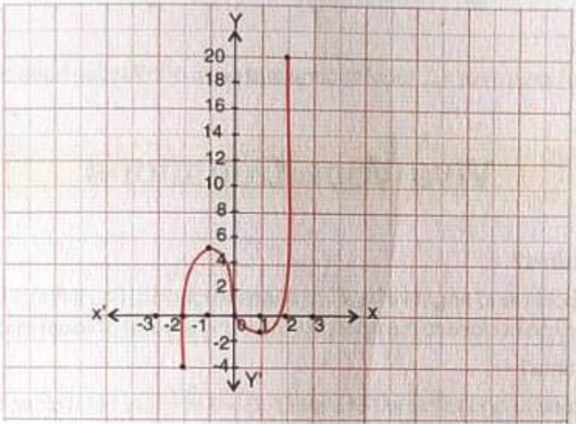

Figure 15

(3) Let us consider a function $f(x) = 3x^3 + 2x^2 - 6x$ in the interval $[-2, 2]$.

(4) We take different values of x in the interval $[-2, 2]$ and find the values of $f(x)$.

Some ordered pairs on the graph of $f(x)$ are as follows:

x	−2	−1	0	1	2
f(x)	−4	5	0	−1	20

(5) By plotting these points on the graph paper and joining the points we obtain the curve of the given function.

(6) Now join these points by free hand and obtain the graph of the function as shown in Fig.15.

OBSERVATIONS :

(1) From the graph, we see that the value of $f(x)$ at $x = 2$ is 20, which is maximum in $[-2, 2]$.

So, **absolute maximum value of f(x) is 20.**

(2) Also, from the graph we see that the value of $f(x)$ at $x = -2$ is -4, which is minimum in $[-2, 2]$.

So, **absolute minimum value of f(x) is −4.**

RESULT :

The above activity demonstrates the concepts of absolute maximum and absolute minimum values of a function in a closed internal through its graph.

MATHEMATICS

TERM-2

ACTIVITY

ACTIVITY 20

OBJECTIVE :

To verify geometrically that $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

PRE-REQUIRED KNOWLEDGE :

Knowledge of vector algebra, addition of vectors, cross product of vectors, etc.

MATERIALS REQUIRED :

A thick board, white paper, a pair of scissors, sketch pen etc.

PROCEDURE :

(1) Take a thick board and paste a white paper on it.

(2) Draw a line segment OA = 10 cm representing $\vec{c}$.

(3) Draw a line segment OB = 8 cm representing $\vec{a}$ such that $\angle AOB = 60°$. Let $\vec{OB} = \vec{a}$.

(4) Draw a line segment BC = 6 cm representing $\vec{b}$ and making angle of 30° with $\vec{OA}$.

(5) Draw BM $\perp$ OA, CL $\perp$ OA and BN $\perp$ CL.

(6) Complete the parallelograms OAQB, OAPC and BQPC.

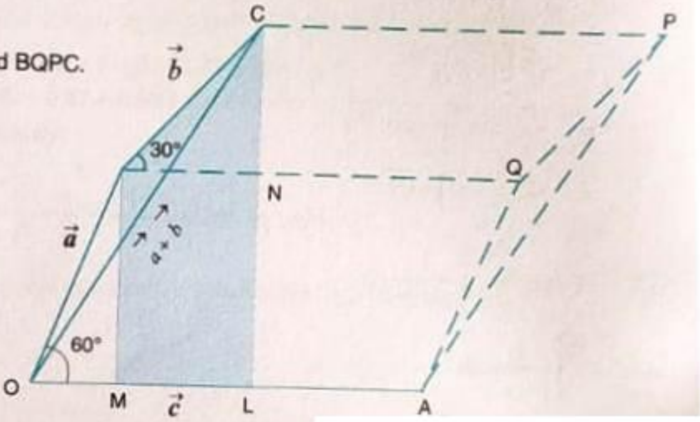

Figure 20

OBSERVATIONS :

(1) $|\vec{c} \times \vec{a}| = |\vec{c}|\,|\vec{a}| \sin 60°$
= OA × BM
= Area of parallelogram OAQB.

(2) $|\vec{c} \times \vec{b}| = |\vec{c}|\,|\vec{b}| \sin 30°$
= OA × CN
= BQ × CN
= Area of parallelogram BQPC.

(3) $\vec{OB} = \vec{a}$ and $\vec{BC} = \vec{b}$.
$\therefore$ In ΔOBC, $\vec{OC} = \vec{OB} + \vec{BC} = \vec{a} + \vec{b}$.
and $\angle COA = \theta$.

(4) $|\vec{c} \times (\vec{a} + \vec{b})| = |\vec{c}|\,|\vec{a} + \vec{b}| \sin \theta$
= OA × OC sin θ = OA × CL
Area of parallelogram OAPC.

(5) Area of the parallelogram OAPC = OA x CL
= OA × (LN + NC)
= OA x (BM + NC)
= OA x BM + OA x NC
=Area of parallelogram OAQB + Area of parallelogram BQPC
$\therefore |\vec{c} \times (\vec{a} + \vec{b})| = |\vec{c} \times \vec{a}| + |\vec{c} \times \vec{b}|$

(6) $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ are perpendicular to the same plane.
$\therefore$ $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$ (condition of co-planarity)

RESULT:

From the above activity it is verified that for any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ we have $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

ACTIVITY 21

OBJECTIVE :
To verify that angle in a semicircle is a right angle, using vector method.

PRE-REQUIRED KNOWLEDGE :
Knowledge of properties of circle and vectors.

MATERIALS REQUIRED :
Plywood, white sheets, nails, threads, glue stick, paper arrowheads etc.

PROCEDURE :

(1) Take a plywood of size 50 × 50 cm and paste a white sheet of paper on it.

(2) On the white sheet of paper, draw a circle of radius 10 cm with centre O.

(3) Draw a diameter DE of this circle.

(4) Take any point A on the circumference of this circle, as shown in Fig.21(a).

(5) Fix nails at O, D, E and A.

(6) Join OD, OE, OA, DA and EA, using thread, stick arrowheads on threads along OD, OE, OA, DA and EA, as shown in Fig.21(a).

Arrowheads show that OD, OE, OA, DA and EA are vectors.

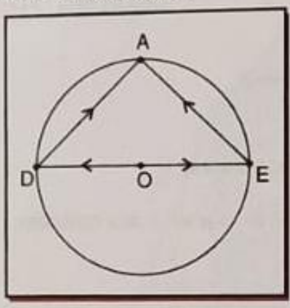

Figure 21 (a)

(7) Now take another plywood of dimensions 50 cm × 50 cm and repeat steps.

(8) Take any two points B and C on the circumference of the circle as shown in Fig.21(b).

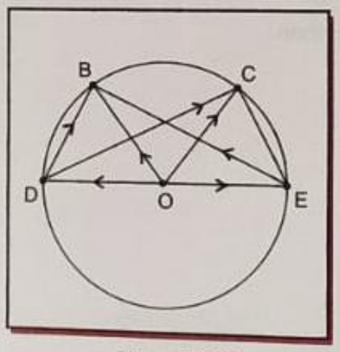

Figure 21 (b)

(9) Fix nails at O, D, E, B and C.

(10) Join OD, OE, OB, OC, DB, DC, EB, EC using threads. Stick arrowheads on threads along OD, OE, OB, OC, DB, DC, EB and EC, as shown in Fig.21(b).These arrowheads are to show them as vectors.

OBSERVATIONS :

(1) By actual measurement from Fig.21(a). We have:

$|\vec{OA}| = 10$ cm, $|\vec{OD}| = 10$ cm, $|\vec{OE}| = 10$ cm,

$|\vec{DA}| = 12$ cm, $|\vec{EA}| = 16$ cm, $|\vec{DE}| = 20$ cm,

$|\vec{DA}|^2 + |\vec{EA}|^2 = 144 + 256 = 400 = |\vec{DE}|^2$

$\Rightarrow \angle DAE = 90°$ [Pythagoras theorem]

$\Rightarrow \vec{DA}.\vec{EA} = |\vec{DA}|\ |\vec{EA}| \cos 90° = 0$

(2) Similarly by actual measurement fromFig.21(b). we have :

$|\vec{OB}| = |\vec{OC}| = |\vec{OD}| = |\vec{OE}| = 10$ cm,

$|\vec{DB}| = 8$ cm, $|\vec{EB}| = 18.3$ cm, $|\vec{DE}| = 20$ cm,

$|\vec{DC}| = 17$ cm, $|\vec{EC}| = 10.5$ cm

$\therefore |\vec{DB}|^2 + |\vec{EB}|^2 = 8^2 + (18.3)^2 \approx 400 = |\vec{DE}|^2$

$\Rightarrow \angle DBE = 90°$ [Pythagoras theorem]

$\Rightarrow \vec{DB}.\vec{EB} = |\vec{DB}||\vec{EB}| \cos 90° = 0.$

Also, $|\vec{DC}|^2 + |\vec{EC}|^2 = 17^2 + (10.5)^2 \approx 400 = |\vec{DE}|^2$

$\Rightarrow \angle DCE = 90°$

$\Rightarrow \vec{DC}.\vec{EC} = |\vec{DC}||\vec{EC}| \cos 90° = 0.$

(3) Also using a protractor, if we measure the angle between the vectors $\vec{DA}$ and $\vec{EA}$, it comes out to be 90° i.e. $\angle DAE = 90°$

Similarly, on measuring angles between the vectors $\vec{DB}$ and $\vec{EB}$ is 90° i.e. $\angle DBE = 90°$ and angle between the vectors $\vec{DC}$ and $\vec{EC}$ is 90° i.e. $\angle DCE = 90°$

RESULT :

From the above activity, it is verified that the angle in a semicircle is a right angle.

ACTIVITY 23

OBJECTIVE :
To demonstrate the equation of the plane in normal form.

PRE-REQUIRED KNOWLEDGE :
Knowledge of vector of a point, plane equation of a plane etc.

MATERIALS REQUIRED :
Two cardboard sheets, wooden sticks, some wires, arrows, glue stick etc.

PROCEDURE :
(1) Take two cardboard sheets each of dimensions 15 cm × 20 cm and fix a wooden stick (ON) between them as shown in Fig.23. Note that the wooden stick ON should be perpendicular to both the cardboard sheets. Here the cardboard sheets represent two planes and the wooden stick represents the normal to the planes and O as the origin.

(2) Now, fix three straight pieces of wires as OA, OB and AB as shown in Fig.23.

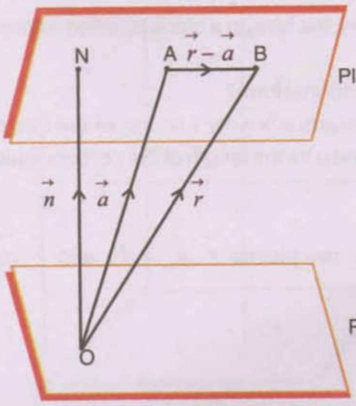

Figure 23

Here B and A are any two points on plane I.

(3) Now stick arrows on wooden stick ON,OA, OB and AB as shown in Fig.23. These arrows show ON, OA, OB and AB as vectors.

OBSERVATIONS :
(1) O is the origin and $\vec{ON}$ is normal to plane I. Let $\vec{ON} = \vec{n}$.

(2) $\vec{a}$ is the position vector of A, $\vec{r}$ is the position vector of P.

So, $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{r}$

$\therefore$ $\vec{OA} + \vec{AB} = \vec{OB}$

$\Rightarrow$ $\vec{AB} = \vec{OB} - \vec{OA} = \vec{r} - \vec{a}$

(3) The vector $\vec{AB} = \vec{r} - \vec{a}$ lies on the plane I and vector $\vec{n}$ is perpendicular to $\vec{r} - \vec{a}$.

So, $(\vec{r} - \vec{a}).\vec{n} = 0$

Hence, $(\vec{r} - \vec{a}).\vec{n} = 0$ is the equation of plane I in normal form.

RESULT :

From the above activity, it is verified that the equation of a plane in normal form is $(\vec{r} - \vec{a}).\vec{n} = 0$

ACTIVITY 26

OBJECTIVE :
To measure the shortest distance between two skew lines and verify it analytically.

PRE-REQUIRED KNOWLEDGE :
Knowledge of equations of a straight line and skew lines, shortest distance between two skew lines.

MATERIALS REQUIRED :
A plywood, graph paper, three wooden blocks of dimensions 1 cm × 1 cm × 1cm &one wooden block of 1 cm × 1 cm × 3 cm, threads, glue stick etc.

PROCEDURE :

(1) Take a plywood of dimensions 25 cm × 15 cm and paste a graph paper on it.

(2) On the graph paper, draw two perpendicular lines OX and OY as x-axis and y-axis respectively.

(3) Locate points A(2, 2), B(7, 2), C(4, 8) and D(11, 9) on the graph paper.

(4) Label the three wooden blocks of dimension 1 cm × 1 cm × 1 cm as a, b and c & label the other wooden block of dimension 1 cm × 1 cm × 3 cm as d.

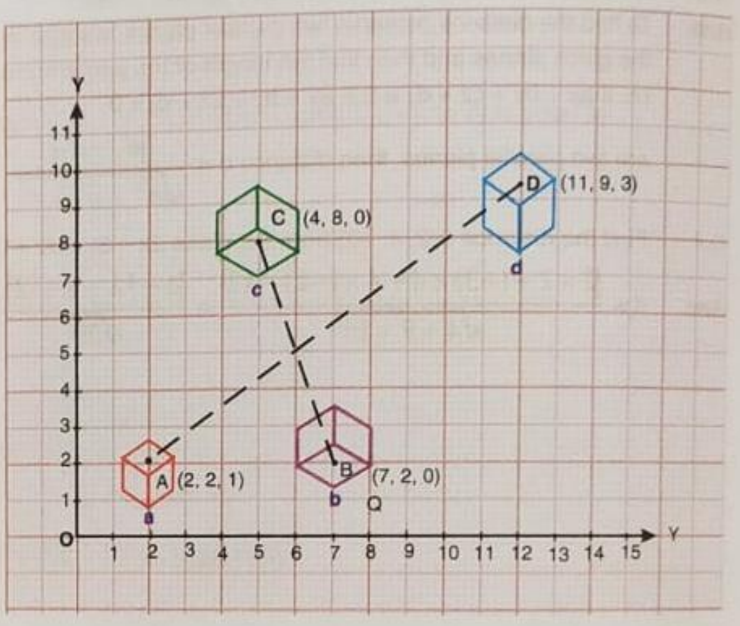

Figure 26

(5) Now fix wooden blocks labeled as a, b and c at A(2, 2), B(7, 2) and C(4, 8) respectively, such that their base centres fall exactly at these points. Fix the wooden labelled as d at D(11, 9) with its centre exactly at (11, 9).

(6) Using gluestick, fix a piece of thread joining points A and B where A and D are centres of the tops of blocks a and d respectively.

(7) Similarly, fix a piece of thread joining points B and C where B and C are centres of bases of blocks b and c respectively.

(8) Take a thread and join it perpendicularly with the lines AD and BC and measure the actual distance.

(9) Place a set square such that its one side forming the right angle is along the thread BC.

(10) Move the set square along AD till its other side forming the right angle touches the thread.

(11) Measure the distance between the two threads in this position to get the required shortest distance between AD and BC.

OBSERVATIONS:

(1) Threads joining AD and BC represent two skew lines.

(2) On actual measurement the shortest distance between the skew lines = 2.5 cm.

(3) Equations of the line joining A(2, 2, 1), and D(11, 9, 3) are given by $\frac{x-2}{11-9}=\frac{y-2}{9-2}=\frac{z-1}{3-1}$

or $\frac{x-2}{2}=\frac{y-2}{7}=\frac{z-1}{2}$ ... (i)

Equations of the lines joining B(7, 2, 0) and C(4, 8, 0) are

$\frac{x-7}{4-7}=\frac{y-2}{8-2}=\frac{z-0}{0-0}$ or $\frac{x-7}{-3}=\frac{y-2}{6}=\frac{z-1}{0}$ ... (ii)

So, the shortest distance d between the lines (i) and (ii) is 3.

(4) We know that the shortest distance d between the lines is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ or

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by

$$d=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}\right|$$

$$d=\left|\frac{\begin{vmatrix} 7-2 & 2-2 & 0-1 \\ 2 & 7 & 2 \\ -3 & 6 & 0 \end{vmatrix}}{\sqrt{(12+21)^2+(0-12)^2+(-6+0)^2}}\right|$$

$$d=\left|\frac{-93}{35.62}\right|=2.61 \text{ cm.}$$

(5) From (3) and (4) we see that the shortest distance between skew lines AD and BC, by actual measurement is approximately equal to the shortest distance obtained analytically.

RESULT :

From the above activity, we observe that the shortest distance between two skew lines obtaine by actual measurement and obtained analytically comes out to be equal.

ACTIVITY 27

OBJECTIVE :

To explain the computation of conditional probability of a given event A when event B has already occurred through an example of throwing a pair of dice.

PRE-REQUIRED KNOWLEDGE :

Knowledge of probability i.e. random experiment, sample space, event, equally likely events, conditional probability etc.

MATERIALS REQUIRED :

A thick board, 36 square sheets of 2 cm × 2 cm, glue stick etc.

PROCEDURE :

(1) Take a thick board paste a squared paper containing 36 squares each of size 2cm × 2cm as shown in Fig. 27.

(2) Write all possible outcomes obtained by throwing two dices on the squared papers.
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Figure 27

OBSERVATIONS :

(1) To find the conditional probability of an event A, when B has already occurred, where A is the event of **a number 3 appears on both the dice and B is the event 3 has already appeared on one of the dice.**
Here, we have to find P(A/B)

From Fig. 27.

Outcome favourable to A is **(3, 3)**

∴ No. of outcomes favourable to A, i.e., n (A) = 1

Outcomes favourable to B are (1, 3), (2, 3), **(3, 3),** (4, 3), (5, 3), (6, 3), (3, 1), (3, 2), (3, 4) (3, 5) (3, 6)

∴ No. of outcomes favourable to B i.e. n(B) = 11

Outcomes which is common to A and B is **(3, 3)**

∴ No. of outcomes favourable to $(A \cap B)$ i.e. n $(A \cap B) = 1$

Hence, P(A/B) = $\frac{n(A \cap B)}{n(B)} = \frac{1}{11}$

Another Method :

We can also use P(A/B) = $\frac{P(A \cap B)}{P(B)}$

Total no. of outcomes = 36

$\therefore$ n(s) = 36

P(B) = $\frac{n(B)}{n(S)} = \frac{11}{36}$

P (A $\cap$ B) = $\frac{n(A \cap B)}{n(S)} = \frac{1}{36}$

So, P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{11}{36}} = \frac{1}{11}$.

(2) To find the conditional probability of an event A, when B has already occurred, where the event **getting a sum 8 and B is the event of a doublet has already occurred.** Here we have to also find P(A/B)

From Fig.27.

Outcomes favaourable to A are (2, 6), (3, 5), **(4, 4)**, (5, 3), (6, 2)

$\therefore$ No. of outcomes favourable to A i.e. n (A) = 5

Outcomes favourable to B are (1, 1), (2, 2), (3, 3), **(4, 4)**, (5, 5), (6, 6).

$\therefore$ No. of outcomes favourable to B i.e. n(B) = 6

Outcome which is common to A and B is **(4, 4)**

$\therefore$ No. of outcomes favourable to (A $\cap$B) i.e. n (A $\cap$B) = 1

Hence, P(A/B) = $\frac{n(A \cap B)}{n(B)} = \frac{1}{6}$.

Another Method:

We can also use P(A/B) = $\frac{P(A \cap B)}{P(B)}$

n(s) = 36

P(B) = $\frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

P (A $\cap$ B) = $\frac{n(A \cap B)}{n(S)} = \frac{1}{36}$

So, P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$.

(3) To find the conditional probability of an event A, when B has already occurred, where A is the event **the sum of the numbers on the two dice is 8 and B is the event numbers appearing on two dice are different.** Here also, we have to find P(A/B).

From Fig.27.

Outcomes favaourable to A are **(2, 6), (3, 5)**, (4, 4),**(5, 3), (6, 2)**

∴ No. of outcomes favourable to A i.e. n (A) = 5

Outcomes favourable to B are(1,2), (1,3), (1, 4), (1, 5),(1,6),(2,1), (2, 3), (2, 4), (2,5),**(2, 6)**, (3,1), (3, 2), (3,4),**(3, 5)**,(3,6),(4, 1),(4,2),(4,3), (4,5), (4,6), (5,1), (5,2),**(5,3)**, (5,4),(5,6), (6,1), **(6,2)**, (6,3), (6,4), (6,5)

∴ No. of outcomes favourable to B i.e. n(B) = 30

Outcomes which are common to A and B are**(2, 6), (3, 5), (5, 3), (6, 2)**

∴ No. of outcomes favourable to (A B) i.e. n(A B) = 4

Hence, P(A/B) = $\frac{n(A \cap B)}{n(B)} = \frac{4}{30} = \frac{2}{15}$

Another Method:

We can also use P(A/B) = $\frac{P(A \cap B)}{P(B)}$

n(s) = 36

P(B) = $\frac{n(B)}{n(S)} = \frac{30}{36} = \frac{15}{18}$

P (A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

So, P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{9}}{\frac{15}{18}} = \frac{2}{15}$.

RESULT :

The above activity explains how to compute the conditional probability of an event, when another event has already occurred.