

# **[CLASS XII- MATHEMATICS]**

**2024-2025**

**Activity No.**

**1, 3, 5, 6, 13, 15, 20, 21, 26, 27**

***Concern Teacher***

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# ACTIVITY 1

## OBJECTIVE:

To verify that the relation  $R$  in the set  $L$  of all lines in a plane, defined by  $R = \{(l, m) : l \perp m\}$  is symmetric but neither reflexive nor transitive.

## PRE-REQUIRED KNOWLEDGE:

Knowledge of relations & properties of parallel lines and perpendicular lines.

## MATERIALS REQUIRED :

A thick board, graph papers, drawing pins, ruler, pencil, glue, colour pens etc.

## PROCEDURE :

- (1) Take a thick board attached a graph paper on it with the help of drawing pins as shown in fig 1 (a).
- (2) Draw two parallel lines  $q_1$  &  $q_2$  with blue colour pen & draw three perpendicular lines  $q_3$ ,  $q_4$  &  $q_5$  with green colour pen.
- (3) Similarly, draw two more parallel lines  $q_6$  &  $q_7$  & a perpendicular line  $q_8$  with red colour pen.
- (4) Since, line  $q_3$  is perpendicular to  $q_1$  &  $q_2$ , line  $q_4$  is perpendicular to  $q_1$  &  $q_2$  and  $q_5$  is perpendicular to  $q_1$  &  $q_2$ . Also, line  $q_8$  is perpendicular on lines  $q_6$  &  $q_7$  as shown in figure 1 (b).
- (5) Since, lines  $q_1$  &  $q_2$  are parallel to each other and  $q_3$  is parallel to  $q_4$ ,  $q_5$  is parallel to  $q_3$  &  $q_4$ . Also,  $q_8$  is parallel to  $q_3$ .
- (6) So,  $(q_1, q_3)$ ,  $(q_2, q_3)$ ,  $(q_1, q_4)$ ,  $(q_2, q_4)$ ,  $(q_1, q_5)$ ,  $(q_2, q_5)$ ,  $(q_6, q_8)$ ,  $(q_7, q_8)$   $R$ .



Figure 1 (a)

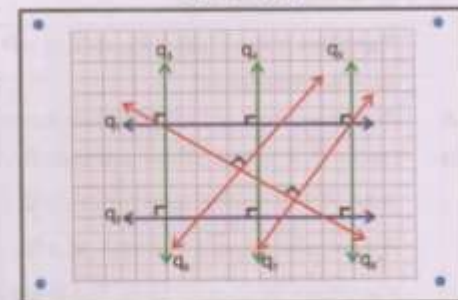


Figure 1 (b)

## OBSERVATIONS :

- (1) In figure 1 (b), we see that  $q_3 \perp q_1$ . Then,  $q_1 \perp q_3$  i.e.  $(q_1, q_3) \in R \Rightarrow (q_3, q_1) \in R$ . Similarly,  $(q_2, q_3) \in R \Rightarrow (q_3, q_2) \in R$  &  $(q_1, q_4) \in R \Rightarrow (q_4, q_1) \in R$ .

So, the given relation  $R$  is symmetric.

- (2) Since, "No line is perpendicular to itself" or  $(l, l) \notin R$

So, the given relation  $R$  is not reflexive.

i.e.  $R = \{(l, m) : l \perp m\}$  is not reflexive.

- (3) It is also observed that  $q_3 \perp q_1$  &  $q_4 \perp q_1$ . But,  $q_3$  is  $\perp$  not on  $q_4$  i.e.  $(q_3, q_1) \in R$  &  $(q_4, q_1) \in R$ . But,  $(q_3, q_4) \notin R$ .

So, the given relation  $R$  is not transitive.

## RESULT :

From the above activity, it is verified that the relation  $R$  in the set  $L$  of all lines in a plane, defined by  $R = \{(l, m) : l \perp m\}$  is symmetric but neither reflexive nor transitive i.e. the relation is not an equivalence relation.

# ACTIVITY 3

## OBJECTIVE :

To demonstrate a function which is not one-one but is onto.

## PRE-REQUIRED KNOWLEDGE :

Basic knowledge of relations, functions and their types (one-one functions and onto functions).

## MATERIALS REQUIRED :

A thick board, coloured chart papers, a pair of scissors, gluestick, some board pins, some pieces of thread.

## PROCEDURE :

- (1) Take a blue chart paper, cut a rectangular piece of length 18 cm and width 5 cm as shown in fig. 3(a).



Figure 3 (a)

- (2) Take a yellow chart paper, cut a rectangular piece of length 15 cm and width 4 cm as shown in Fig. 3(b).

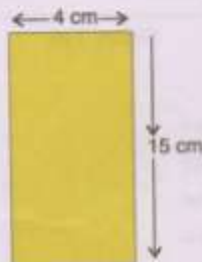


Figure 3 (b)

- (3) Paste these two pieces on a thick board and mark these as points P & Q respectively as shown in Fig.3(c).

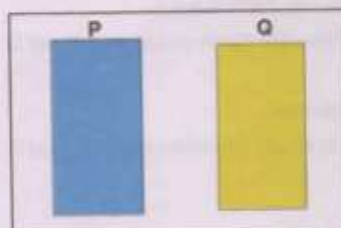


Figure 3 (c)

- (4) Fix five drawing pins on the blue paper and mark the pins as a, b, c, d & e. Similarly, fix three drawing pins on yellow paper and mark them as x, y and z as shown in Fig.3 (d) i.e.  $P = \{a, b, c, d, e\}$  &  $Q = \{x, y, z\}$

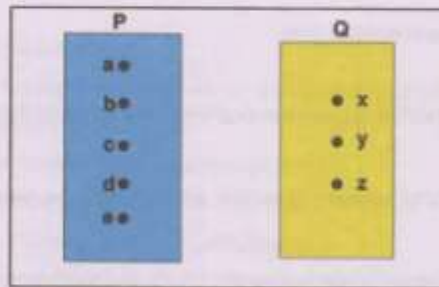


Figure 3 (d)

- (5) Join these drawing pins with the help of threads as shown in Fig.3(e) i.e. join the elements of P to the elements of Q.

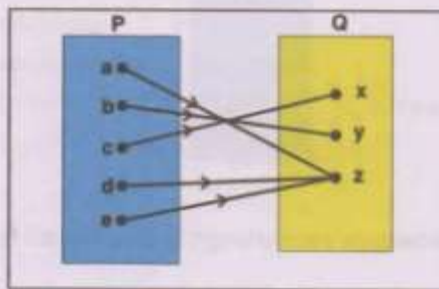


Figure 3 (e)

### OBSERVATIONS :

- (1) The image of the element a of set P in set Q is z.
- (2) The image of the element b of set P in set Q is y.
- (3) The image of the element c of set P in set Q is x.
- (4) The image of the element d of set P in set Q is z.
- (5) The image of the element e of set P in set Q is z.
- (6) The pre-image of the element x of set Q in set P is c.
- (7) The pre-image of the element y of set Q in set P is b.
- (8) The pre-image of the element z of set Q in set P are a, d & e.
- (9) Since, the elements of a, d & e in set P have the same image as z in set Q i.e. every element of set P has not one-one image in set Q.

**So, the function is many-one or not one-one.**

- (10) Also, we see that every element of set Q is image of every element of set P i.e. the pre-image of every element of Q in P exists.

**So, the function is onto.**

### RESULT :

From the above activity we can say that the function is **not one-one but onto.**

# ACTIVITY 5

## OBJECTIVE :

To draw the graph of  $\sin^{-1}x$ , using the graph of  $\sin x$  and demonstrate the concept of mirror reflection (about the line  $y = x$ ).

## PRE-REQUIRED KNOWLEDGE :

- (i) Knowledge of plotting the graph of  $\sin x$ .
- (ii) Knowledge of trigonometric functions and inverse of trigonometric functions.

## MATERIALS REQUIRED :

A thick board of dimensions 20 cm × 20 cm, ruler, coloured pencils, chart paper, drawing pins and strings.

## PROCEDURE :

- (1) Paste a chart paper & on a thick board with the help of drawing pins as shown in Fig.5(a).

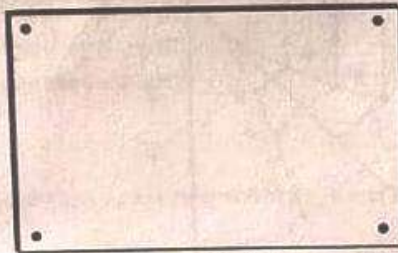


Figure 5 (a)

- (2) On the chart paper, draw perpendicular axes  $XOX'$  and  $YOY'$  as shown in Fig.5(b).

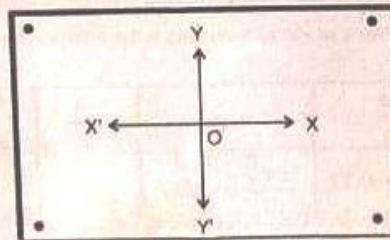


Figure 5 (b)

- (3) Mark the points on y-axis as  $1, \frac{1}{2}, 0, \frac{-1}{2}, -1$ , etc.
- (4) Plot the graph of  $y = \sin x$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2} = 0.5$	$\frac{1}{\sqrt{2}} = 0.71$	$\frac{\sqrt{3}}{2} = 0.87$	1

- (5) Fix drawing pins in the coordinate plane to represent the points namely  $A \left( \frac{\pi}{6}, 0.5 \right)$ ,  $B \left( \frac{\pi}{4}, 0.71 \right)$ ,  $C \left( \frac{\pi}{3}, 0.87 \right)$  &  $D \left( \frac{\pi}{2}, 1 \right)$  as shown in Fig.5(c).

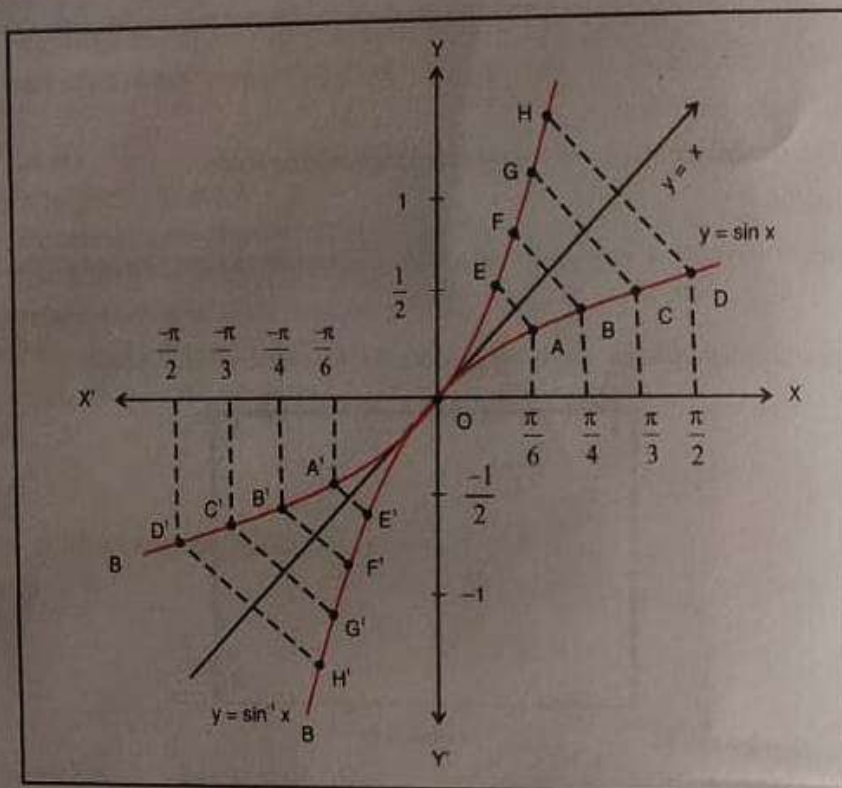


Figure 5 (c)

- (6) Similarly, on the x-axis, repeat the same procedure and mark the points in the table below:

$x$	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$	$-\frac{\pi}{2}$
$\sin x$	$\frac{-1}{2} = -0.5$	$\frac{-1}{\sqrt{2}} = -0.71$	$\frac{-\sqrt{3}}{2} = -0.87$	$-1$

- (7) Fix the drawing pins on the points namely  $A' \left( \frac{-\pi}{6}, -0.5 \right)$ ,  $B' \left( \frac{-\pi}{4}, -0.71 \right)$ ,  $C' \left( \frac{-\pi}{3}, -0.87 \right)$ ,  $D' \left( \frac{-\pi}{2}, -1 \right)$  & fix one drawing pin on the origin O, as shown in Fig.5 (c).

- (8) Join the pins with the help of a string on both sides of x-axis. Here, we will get a curve which is the graph of  $\sin x$  from  $\frac{-\pi}{2}$  to  $\frac{\pi}{2}$ .

- (9) Now plot the points  $(1, 1), (2, 2), (3, 3), (4, 4), \dots$  etc. on the coordinate plane to draw the graph of line  $y = x$ .

- (10) From the marked points A, B, C, D draw perpendiculars on the line  $y = x$  and construct lines such that length of

perpendicular on both sides of the line  $y = x$  are equal. Mark these points as E, F, G, H and fix the pins on them.

- (11) Repeat the same process on the other side of  $x$ -axis and fix the pins on the points namely  $E', F', G', H'$ .
- (12) Join these pins on both sides of the line  $y = x$  by a string gently to obtain the graph of  $y = \sin^{-1}x$ .
- (13) Now place a mirror on the line  $y = x$ . The mirror image of the graph of  $\sin x$  represents the graph of  $\sin^{-1}x$ , which shows  $\sin^{-1}x$  is a reflection of  $\sin x$  about the line  $y = x$ .

### OBSERVATIONS :

- (1) The image of point A in the mirror (the line  $y = x$ ) is E.
- (2) The image of point B in the mirror (the line  $y = x$ ) is F.
- (3) The image of point C in the mirror (the line  $y = x$ ) is G.
- (4) The image of point D in the mirror (the line  $y = x$ ) is H.
- (5) The image of point  $A'$  in the mirror (the line  $y = x$ ) is  $E'$ .
- (6) The image of point  $B'$  in the mirror (the line  $y = x$ ) is  $F'$ .
- (7) The image of point  $C'$  in the mirror (the line  $y = x$ ) is  $G'$ .
- (8) The image of point  $D'$  in the mirror (the line  $y = x$ ) is  $H'$ .

### RESULT :

The mirror image of the graph of  $\sin x$  about the line  $y = x$  is the graph of  $\sin^{-1}x$  and the mirror image of the graph of  $\sin^{-1}x$  about the line  $y = x$  is the graph of  $\sin x$ .

# ACTIVITY 6

## OBJECTIVE :

To explore the principal value of the function  $\sin^{-1}x$  using a unit circle.

## PRE-REQUIRED KNOWLEDGE :

Basic knowledge & properties of trigonometric ratios and inverse trigonometric functions.

## MATERIALS REQUIRED :

A thick board, white paper, drawing pins, two sticks, ruler, needle and wires.

## PROCEDURE :

- (1) Take a white paper on a thick board with the help of drawing pins as shown in Fig.6(a).

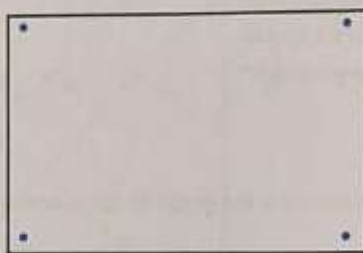


Figure 6 (a)

- (2) Take radius = 1 unit and draw a circle with centre O on it.
- (3) Through the centre of the circle draw perpendicular axes  $XOX'$  and  $YOY'$  as x-axis and y-axis respectively as shown in Fig. 6(b).

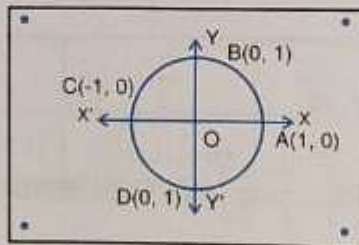


Figure 6 (b)

- (4) Mark the points A, B, C and D, where the circle cuts the x-axis and y-axis respectively as shown in Fig. 6(b).

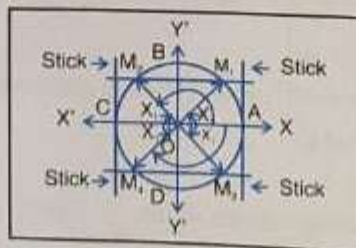


Figure 6 (c)



- (5) Fix two sticks parallel to y-axis on the opposite sides of the board as shown in Fig.6(c).

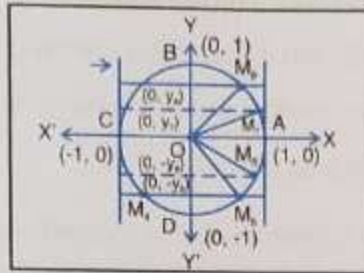


Figure 6 (d)

- (6) Now fix one wire between the sticks so that the wire can move freely parallel to the x-axis as shown in Fig. 6(c).
- (7) Now fix a needle of unit length such that one end of the needle is at the centre of the circle and the other end to move freely along the circle as shown in Fig. 6(c).
- (8) Place the needle at an arbitrary angle  $x$ , with the positive x-axis.
- (9) Measurement of the angle in radian is equal to the length of intercepted arc of the circle.
- (10) Slide the wire between the sticks such that the wire meets with free end of the needle. Mark this point as  $M_1$ , as shown in Fig.6(c).
- (11) Identify the y-coordinate of point  $M_1$ , which is equal to the perpendicular distance from the x-axis. It gives  $y_1 = \sin x_1$ .
- (12) Further rotate the needle in the anticlockwise direction and keep it at the angle  $\pi - x_1$ . Here the wire meets the needle at point  $M_2$  as shown in Fig.6(c).
- (13) With the help of sliding wire, find the value of y-coordinate of  $M_2$ .
- (14) For both the points  $M_1$  and  $M_2$  the value of y-coordinates are same for different values of angles i.e.  $y_1 = \sin x_1$  and  $y_1 = \sin(\pi - x_1)$ .
- (15) The above observations show that the sine function is not one-to-one for angles which lie in first and second quadrants.
- (16) Repeat the same process for angles  $-x_1$  and  $(-\pi + x_1)$  respectively. In this case also we will get the same result that y-coordinate for the points  $M_3$  and  $M_4$  are the same. Thus, the sine function is not one-to-one for angles that fall in third and fourth quadrant, as shown in Fig.6(c).
- (17) Here, we observe that the value of y-coordinate is different for points  $M_1$  and  $M_2$ .
- (18) Now move the needle again in anti clockwise direction from  $-\frac{\pi}{2}$  to  $\frac{\pi}{2}$ . By sliding the wire, we see that the value of y-coordinate for points  $M_5, M_6, M_7$  and  $M_8$  are different. Therefore, sine function is one-to-one in the domain  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and thus its range lies between -1 and 1, as shown in Fig.6(d).

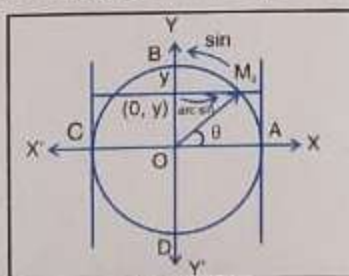


Figure 6 (e)

(19) Now place the needle at any arbitrary angle  $\theta$  lying in the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and find the y-coordinate of the intersecting point  $M$ , through the sliding wire. We call it as  $y$ .  
 $y = \sin \theta$  or  $\theta = \sin^{-1}y$  as sine function is one-one and onto in the domain  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and range  $[-1, 1]$ .  
Hence, its inverse arc sine function exists as shown in Fig. 6(e).

(20) The domain and range of sine inverse function are interchanged with the domain and range of sine function, i.e., the domain of arc sine function is  $[-1, 1]$  and range is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

This range of arc sine function is known as principal value of arc sine function or  $\sin^{-1}$  function.

### OBSERVATIONS :

- (1) Sine function is non-negative in first and second quadrants.
- (2) For the third and fourth quadrants, the sine function is negative.
- (3)  $\theta = \arcsin y \Rightarrow y = \sin^{-1} \theta$ , where  $\left[-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right]$

### RESULT :

From the above activity, we find that the principal value range of  $\sin^{-1} x$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

# ACTIVITY 13

## OBJECTIVE :

To understand the concepts of decreasing and increasing functions.

## PRE-REQUIRED KNOWLEDGE :

Knowledge of increasing and decreasing function and their properties.

## MATERIALS REQUIRED :

1 metre G.I. (Galvanized iron wire), rectangular & square pieces of cardboard, white paper, gluestick, trigonometric table book or scientific calculator etc.

## PROCEDURE :

- (1) Take a square cardboard and paste a white paper on it.
- (2) Take two pieces of wire of 15 cm length and fix them on the white paper to represent x-axis and y-axis.
- (3) Take two more pieces of wire each of 10 cm length and bend them in the shape of curves representing two functions and fix them on the paper to represent two functions, as shown in Fig. 13.

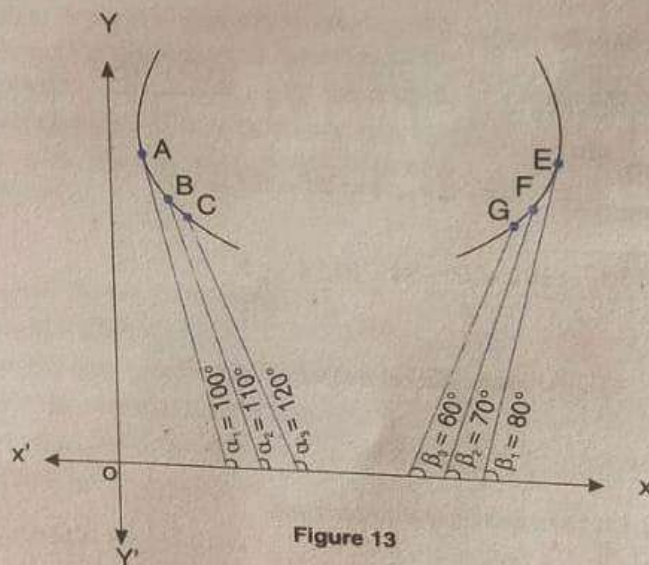


Figure 13

- (4) Take one straight wire and place it on the curve (on the left) such that it is tangent to the curve at the point say A and making an angle  $\alpha_1$  with the positive direction of x-axis.
- (5) Take two more points say B and C on the same curve and make tangents, using the same wire at B and C making angles  $\alpha_2$  and  $\alpha_3$ , respectively with the positive direction of x-axis.
- (6) Now take three more points E, F, G on the curve (on the right) and using the other straight wires, form tangents at each of these points making angles  $\beta_1, \beta_2, \beta_3$ , respectively with the positive direction of x-axis as shown in Fig. 13.

## OBSERVATIONS :

- (1)  $\alpha_1 = 100^\circ$  is an obtuse angle, so  $\tan \alpha_1$  is negative, i.e., the slope of the tangent at A is negative.

(2) Again,  $\alpha_2 = 110^\circ$  and  $\alpha_3 = 120^\circ$  are obtuse angles and therefore, slopes of the tangents  $\tan \alpha_2$  and  $\tan \alpha_3$  are both negative.

$$\begin{aligned} \Rightarrow \tan \alpha_1 &= \tan 100^\circ = -5.6712 \\ \Rightarrow \tan \alpha_2 &= \tan 110^\circ = -2.7474 \\ \Rightarrow \tan \alpha_3 &= \tan 120^\circ = -1.7320 \end{aligned} \left. \vphantom{\begin{aligned} \Rightarrow \tan \alpha_1 \\ \Rightarrow \tan \alpha_2 \\ \Rightarrow \tan \alpha_3 \end{aligned}} \right\} \text{These are negative values.}$$

The function given by the curve (on the left) is a decreasing function.

(3) On measuring  $\beta_1 = 80^\circ$ ,  $\beta_2 = 70^\circ$ , and  $\beta_3 = 60^\circ$ .

$$\begin{aligned} \Rightarrow \tan \beta_1 &= \tan 80^\circ = +5.6712 \\ \Rightarrow \tan \beta_2 &= \tan 70^\circ = +2.7474 \\ \Rightarrow \tan \beta_3 &= \tan 60^\circ = +1.7320 \end{aligned} \left. \vphantom{\begin{aligned} \Rightarrow \tan \beta_1 \\ \Rightarrow \tan \beta_2 \\ \Rightarrow \tan \beta_3 \end{aligned}} \right\} \text{These are positive values.}$$

The function given by the curve (on the right) is an increasing function.

### RESULT :

From the above activity, it is verified that a function  $f(x)$  is increasing when  $f'(x) \geq 0$  and decreasing when  $f'(x) \leq 0$ , (where  $x \in \text{Domain } f$ ).

# ACTIVITY 15

## OBJECTIVE :

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

## PRE-REQUIRED KNOWLEDGE :

Knowledge of maxima, minima, absolute maxima & absolute minimum.

## MATERIALS REQUIRED :

Thick board, white chart paper, sketch pens, calculator, glue stick etc.

## PROCEDURE :

- (1) Paste a white chart paper on the thick board.
- (2) Draw two lines on the graph paper representing x-axis and y-axis as shown in the Fig. 15.

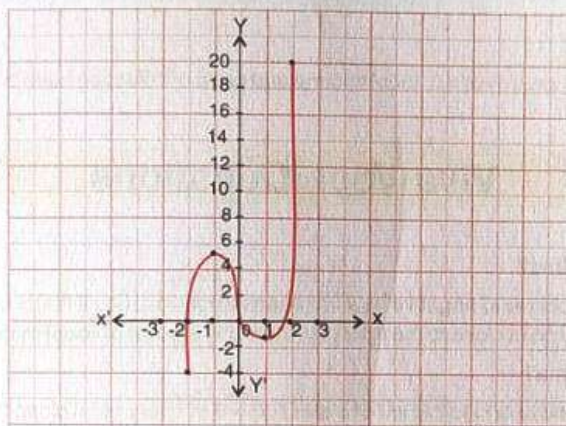


Figure 15

- (3) Let us consider a function  $f(x) = 3x^3 + 2x^2 - 6x$  in the interval  $[-2, 2]$ .
- (4) We take different values of  $x$  in the interval  $[-2, 2]$  and find the values of  $f(x)$ .  
Some ordered pairs on the graph of  $f(x)$  are as follows:

x	-2	-1	0	1	2
f(x)	-4	5	0	-1	20

- (5) By plotting these points on the graph paper and joining the points we obtain the curve of the given function.
- (6) Now join these points by free hand and obtain the graph of the function as shown in Fig. 15.

## OBSERVATIONS :

- (1) From the graph, we see that the value of  $f(x)$  at  $x = 2$  is 20, which is maximum in  $[-2, 2]$ .  
So, **absolute maximum value of  $f(x)$  is 20.**
- (2) Also, from the graph we see that the value of  $f(x)$  at  $x = -2$  is -4, which is minimum in  $[-2, 2]$ .  
So, **absolute minimum value of  $f(x)$  is -4.**

## RESULT :

The above activity demonstrates the concepts of absolute maximum and absolute minimum values of a function in a closed interval through its graph.

# ACTIVITY 20

## OBJECTIVE :

To verify geometrically that  $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

## PRE-REQUIRED KNOWLEDGE :

Knowledge of vector algebra, addition of vectors, cross product of vectors, etc.

## MATERIALS REQUIRED :

A thick board, white paper, a pair of scissors, sketch pen etc.

## PROCEDURE :

- (1) Take a thick board and paste a white paper on it.
- (2) Draw a line segment  $OA = 10$  cm representing  $\vec{c}$ .
- (3) Draw a line segment  $OB = 8$  cm representing  $\vec{a}$  such that  $\angle AOB = 60^\circ$ . Let  $\vec{OB} = \vec{a}$ .
- (4) Draw a line segment  $BC = 6$  cm representing  $\vec{b}$  and making angle of  $30^\circ$  with  $\vec{OA}$ .
- (5) Draw  $BM \perp OA$ ,  $CL \perp OA$  and  $BN \perp CL$ .
- (6) Complete the parallelograms  $OAQB$ ,  $OAPC$  and  $BQPC$ .

## OBSERVATIONS :

- (1)  $|\vec{c} \times \vec{a}| = |\vec{c}| |\vec{a}| \sin 60^\circ$   
 $= OA \times BM$   
 $= \text{Area of parallelogram } OAQB.$
- (2)  $|\vec{c} \times \vec{b}| = |\vec{c}| |\vec{b}| \sin 30^\circ$   
 $= OA \times CN$   
 $= BQ \times CN$   
 $= \text{Area of parallelogram } BQPC.$
- (3)  $\vec{OB} = \vec{a}$  and  $\vec{BC} = \vec{b}$ .  
 $\therefore$  In  $\triangle OBC$ ,  $\vec{OC} = \vec{OB} + \vec{BC} = \vec{a} + \vec{b}$ .  
 and  $\angle COA = \theta$ .
- (4)  $|\vec{c} \times (\vec{a} + \vec{b})| = |\vec{c}| |\vec{a} + \vec{b}| \sin \theta$   
 $= OA \times OC \sin \theta = OA \times CL$   
 $= \text{Area of parallelogram } OAPC.$
- (5) Area of the parallelogram  $OAPC = OA \times CL$   
 $= OA \times (LN + NC)$   
 $= OA \times (BM + NC)$   
 $= OA \times BM + OA \times NC$   
 $= \text{Area of parallelogram } OAQB + \text{Area of parallelogram } BQPC$   
 $\therefore |\vec{c} \times (\vec{a} + \vec{b})| = |\vec{c} \times \vec{a}| + |\vec{c} \times \vec{b}|$

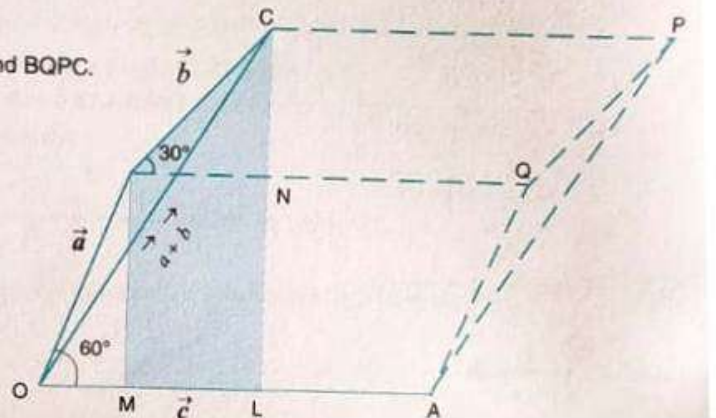


Figure 20

(6)  $\vec{c} \times (\vec{a} + \vec{b})$ ,  $\vec{c} \times \vec{a}$  and  $\vec{c} \times \vec{b}$  are perpendicular to the same plane.

$$\therefore \vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b} \text{ (condition of co-planarity)}$$

**RESULT:**

From the above activity it is verified that for any three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  we have  $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$ .

# ACTIVITY 21

**OBJECTIVE :**

To verify that angle in a semicircle is a right angle, using vector method.

**PRE-REQUIRED KNOWLEDGE :**

Knowledge of properties of circle and vectors.

**MATERIALS REQUIRED :**

Plywood, white sheets, nails, threads, glue stick, paper arrowheads etc.

**PROCEDURE :**

- (1) Take a plywood of size 50 × 50 cm and paste a white sheet of paper on it.
- (2) On the white sheet of paper, draw a circle of radius 10 cm with centre O.
- (3) Draw a diameter DE of this circle.
- (4) Take any point A on the circumference of this circle, as shown in Fig.21 (a).
- (5) Fix nails at O, D, E and A.
- (6) Join OD, OE, OA, DA and EA, using thread, stick arrowheads on threads along OD, OE, OA, DA and EA, as shown in Fig.21(a).  
Arrowheads show that OD, OE, OA, DA and EA are vectors.

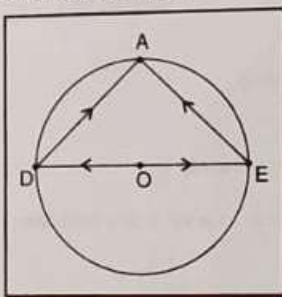


Figure 21 (a)

- (7) Now take another plywood of dimensions 50 cm × 50 cm and repeat steps.
- (8) Take any two points B and C on the circumference of the circle as shown in Fig.21(b).

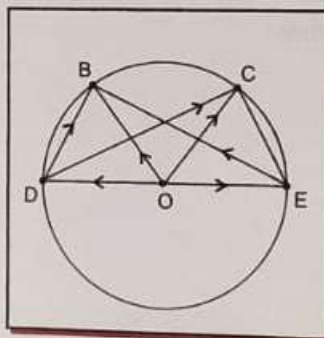


Figure 21 (b)

- (9) Fix nails at O, D, E, B and C.
- (10) Join OD, OE, OB, OC, DB, DC, EB, EC using threads. Stick arrowheads on threads along OD, OE, OB, OC, DB, DC, EB and EC, as shown in Fig.21(b). These arrowheads are to show them as vectors.

## OBSERVATIONS :

(1) By actual measurement from Fig.21(a). We have :

$$|\vec{OA}| = 10 \text{ cm}, |\vec{OD}| = 10 \text{ cm}, |\vec{OE}| = 10 \text{ cm},$$

$$|\vec{DA}| = 12 \text{ cm}, |\vec{EA}| = 16 \text{ cm}, |\vec{DE}| = 20 \text{ cm},$$

$$|\vec{DA}|^2 + |\vec{EA}|^2 = 144 + 256 = 400 = |\vec{DE}|^2$$

$$\Rightarrow \angle DAE = 90^\circ \text{ [Pythagoras theorem]}$$

$$\Rightarrow \vec{DA} \cdot \vec{EA} = |\vec{DA}| |\vec{EA}| \cos 90^\circ = 0$$

(2) Similarly by actual measurement from Fig.21(b). we have :

$$|\vec{OB}| = |\vec{OC}| = |\vec{OD}| = |\vec{OE}| = 10 \text{ cm},$$

$$|\vec{DB}| = 8 \text{ cm}, |\vec{EB}| = 18.3 \text{ cm}, |\vec{DE}| = 20 \text{ cm},$$

$$|\vec{DC}| = 17 \text{ cm}, |\vec{EC}| = 10.5 \text{ cm}$$

$$\therefore |\vec{DB}|^2 + |\vec{EB}|^2 = 8^2 + (18.3)^2 \approx 400 = |\vec{DE}|^2$$

$$\Rightarrow \angle DBE = 90^\circ \text{ [Pythagoras theorem]}$$

$$\Rightarrow \vec{DB} \cdot \vec{EB} = |\vec{DB}| |\vec{EB}| \cos 90^\circ = 0.$$

$$\text{Also, } |\vec{DC}|^2 + |\vec{EC}|^2 = 17^2 + (10.5)^2 \approx 400 = |\vec{DE}|^2$$

$$\Rightarrow \angle DCE = 90^\circ$$

$$\Rightarrow \vec{DC} \cdot \vec{EC} = |\vec{DC}| |\vec{EC}| \cos 90^\circ = 0.$$

(3) Also using a protractor, if we measure the angle between the vectors  $\vec{DA}$  and  $\vec{EA}$ , it comes out to be  $90^\circ$  i.e.  $\angle DAE = 90^\circ$

Similarly, on measuring angles between the vectors  $\vec{DB}$  and  $\vec{EB}$  is  $90^\circ$  i.e.  $\angle DBE = 90^\circ$  and angle between the vectors  $\vec{DC}$  and  $\vec{EC}$  is  $90^\circ$  i.e.  $\angle DCE = 90^\circ$

## RESULT :

From the above activity, it is verified that the angle in a semicircle is a right angle.



# ACTIVITY 26

## OBJECTIVE :

To measure the shortest distance between two skew lines and verify it analytically.

## PRE-REQUIRED KNOWLEDGE :

Knowledge of equations of a straight line and skew lines, shortest distance between two skew lines.

## MATERIALS REQUIRED :

A plywood, graph paper, three wooden blocks of dimensions  $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$  & one wooden block of  $1\text{ cm} \times 1\text{ cm} \times 3\text{ cm}$ , threads, glue stick etc.

## PROCEDURE :

- (1) Take a plywood of dimensions  $25\text{ cm} \times 15\text{ cm}$  and paste a graph paper on it.
- (2) On the graph paper, draw two perpendicular lines OX and OY as x-axis and y-axis respectively.
- (3) Locate points A(2, 2), B(7, 2), C(4, 8) and D(11, 9) on the graph paper.
- (4) Label the three wooden blocks of dimension  $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$  as a, b and c & label the other wooden block of dimension  $1\text{ cm} \times 1\text{ cm} \times 3\text{ cm}$  as d.

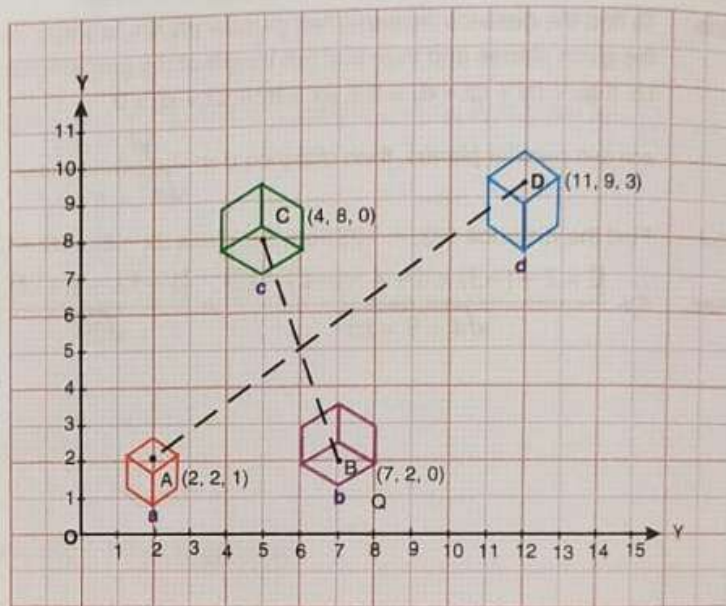


Figure 26

- (5) Now fix wooden blocks labeled as a, b and c at A(2, 2), B(7, 2) and C(4, 8) respectively, such that their base centres fall exactly at these points. Fix the wooden labelled as d at D(11, 9) with its centre exactly at (11, 9).
- (6) Using gluestick, fix a piece of thread joining points A and B where A and D are centres of the tops of blocks a and d respectively.
- (7) Similarly, fix a piece of thread joining points B and C where B and C are centres of bases of blocks b and c respectively.
- (8) Take a thread and join it perpendicularly with the lines AD and BC and measure the actual distance.
- (9) Place a set square such that its one side forming the right angle is along the thread BC.
- (10) Move the set square along AD till its other side forming the right angle touches the thread.
- (11) Measure the distance between the two threads in this position to get the required shortest distance between AD and BC.

## OBSERVATIONS:

- (1) Threads joining AD and BC represent two skew lines.
- (2) On actual measurement the shortest distance between the skew lines = 2.5 cm.

(3) Equations of the line joining A(2, 2, 1), and D(11, 9, 3) are given by  $\frac{x-2}{11-9} = \frac{y-2}{9-2} = \frac{z-1}{3-1}$

$$\text{or } \frac{x-2}{2} = \frac{y-2}{7} = \frac{z-1}{2} \quad \dots \text{(i)}$$

Equations of the lines joining B(7, 2, 0) and C(4, 8, 0) are

$$\frac{x-7}{4-7} = \frac{y-2}{8-2} = \frac{z-0}{0-0} \text{ or } \frac{x-7}{-3} = \frac{y-2}{6} = \frac{z-1}{0} \quad \dots \text{(ii)}$$

So, the shortest distance  $d$  between the lines (i) and (ii) is 3.

(4) We know that the shortest distance  $d$  between the lines is  $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$  or

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1 b_2 - a_2 b_1)^2 + (b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2}}$$

$$d = \frac{\begin{vmatrix} 7-2 & 2-2 & 0-1 \\ 2 & 7 & 2 \\ -3 & 6 & 0 \end{vmatrix}}{\sqrt{(12+21)^2 + (0-12)^2 + (-6+0)^2}}$$

$$d = \frac{-93}{35.62} = 2.61 \text{ cm.}$$

(5) From (3) and (4) we see that the shortest distance between skew lines AD and BC, by actual measurement is approximately equal to the shortest distance obtained analytically.

#### RESULT:

From the above activity, we observe that the shortest distance between two skew lines obtained by actual measurement and obtained analytically comes out to be equal.

# ACTIVITY 27

## OBJECTIVE :

To explain the computation of conditional probability of a given event A when event B has already occurred through an example of throwing a pair of dice.

## PRE-REQUIRED KNOWLEDGE :

Knowledge of probability i.e. random experiment, sample space, event, equally likely events, conditional probability etc.

## MATERIALS REQUIRED :

A thick board, 36 square sheets of 2 cm × 2 cm, glue stick etc.

## PROCEDURE :

- (1) Take a thick board paste a squared paper containing 36 squares each of size 2cm × 2cm as shown in Fig. 27.
- (2) Write all possible outcomes obtained by throwing two dice on the squared papers.  
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Figure 27

## OBSERVATIONS :

- (1) To find the conditional probability of an event A, when B has already occurred, where A is the event of a number 3 appears on both the dice and B is the event 3 has already appeared on one of the dice. Here, we have to find  $P(A/B)$

From Fig. 27.

Outcome favourable to A is (3, 3)

∴ No. of outcomes favourable to A, i.e.,  $n(A) = 1$

Outcomes favourable to B are (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6)

∴ No. of outcomes favourable to B i.e.  $n(B) = 11$

Outcomes which is common to A and B is (3, 3)

∴ No. of outcomes favourable to  $(A \cap B)$  i.e.  $n(A \cap B) = 1$

$$\text{Hence, } P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{11}$$

**Another Method :**

$$\text{We can also use } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Total no. of outcomes = 36

$$\therefore n(s) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$$\text{So, } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{11}{36}} = \frac{1}{11}$$

- (2) To find the conditional probability of an event A, when B has already occurred, where the event **getting a sum 8** and B is the event of a doublet has already occurred. Here we have to also find

**From Fig.27.**

Outcomes favourable to A are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

$\therefore$  No. of outcomes favourable to A i.e.  $n(A) = 5$

Outcomes favourable to B are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).

$\therefore$  No. of outcomes favourable to B i.e.  $n(B) = 6$

Outcome which is common to A and B is (4, 4)

$\therefore$  No. of outcomes favourable to  $(A \cap B)$  i.e.  $n(A \cap B) = 1$

$$\text{Hence, } P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{6}$$

**Another Method:**

$$\text{We can also use } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$n(s) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$$\text{So, } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

- (3) To find the conditional probability of an event A, when B has already occurred, where A is the event the sum of the numbers on the two dice is 8 and B is the event numbers appearing on two dice are different. Here also, we have to find  $P(A/B)$ .

From Fig.27.

Outcomes favourable to A are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

$\therefore$  No. of outcomes favourable to A i.e.  $n(A) = 5$

Outcomes favourable to B are (1,2), (1,3), (1, 4), (1, 5), (1,6), (2,1), (2, 3), (2, 4), (2,5), (2, 6), (3,1), (3, 2), (3,4), (3, 5), (3,6), (4, 1), (4,2), (4,3), (4,5), (4,6), (5, 1), (5,2), (5,3), (5,4), (5,6), (6,1), (6, 2), (6,3), (6,4), (6,5)

$\therefore$  No. of outcomes favourable to B i.e.  $n(B) = 30$

Outcomes which are common to A and B are (2, 6), (3, 5), (5, 3), (6, 2)

$\therefore$  No. of outcomes favourable to (A  $\cap$  B) i.e.  $n(A \cap B) = 4$

$$\text{Hence, } P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{30} = \frac{2}{15}$$

**Another Method:**

$$\text{We can also use } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$n(s) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{36} = \frac{5}{6}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

$$\text{So, } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{9}}{\frac{5}{6}} = \frac{2}{15}$$

#### RESULT :

The above activity explains how to compute the conditional probability of an event, when another event has already occurred.

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